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PROTON-PROTON INTERACTION AND YUKAWA PARTICLE*

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ABSTRACT. A rigorous formula is derived for proton-proton scattering taking the interaction potential to be of Yukawa type, *viz.*, $V(r)=\frac{A}{r}e^{-\alpha r}$. The formula agrees remarkably well with the experiment of Heydenburg, Tuve and Hafstad. It is shown that the fitting values of A and α give the values of the short range charge and mass of 'neutretto'. The charge and mass so obtained are found to be exactly same as those of mesotron previously determined by Heitler from the binding energy of deuteron

In a recent paper [1] I have developed the theory of proton-proton scattering taking the short range potential to be of the type $Ae^{-\alpha r}$. It may be mentioned that a similar short range potential was assumed in the case of neutron-proton interaction [2] in order to explain the spherical symmetry in scattering observed by many experimenters.[3] The point which appears to me to be in favour of taking such a potential is that it gives a constant value at $r=0$. This is just what is found within the hard core in case of spontaneous and artificial disintegrations of atomic nuclei.[4]

In the present article I propose to study the problem of proton-proton scattering taking the short range interaction potential in the form

$$V=\frac{A}{r}e^{-\alpha r}\,, \qquad (1)$$

as suggested by Yukawa.[6] It gives, as before, spherical symmetry in neutron-proton scattering. Furthermore, as $\sqrt{A}$ has the dimension of electrical charge, its fitting value for proton-proton scatttering at a given angle and given incident velocity, gives the value of the electrical charge giving rise to the close range force during proton-proton interaction.

Before going into the details of these discussions I first proceed to derive the scattering formula with Yukawa potential and explain how the formula is verified experimentally. It should be noted at the outset that, as in electron scattering by nucleus,[7] I shall throughout take the solution to be bounded and shall use the wave-statistical idea of critical approach.

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It has been shown before [1, 2] that referring the motion to the centre of mass (C-system) we have for the first order scattering function

$$\lambda_1\chi_1(r_2, \theta_2) = -\frac{1}{4\pi\sqrt{v}} \cdot \frac{k^2}{E} \int V(r_1) e^{ikx_1} \frac{e^{ikr_{12}}}{r_{12}} d\tau_2 \quad \dots \quad (2)$$

where $V(r_1)$ is the perturbing potential, v the velocity of the proton, $E = \frac{1}{4}Mv^2$, M being the proton mass and $k = \frac{\pi M v}{h}$. Now, on taking the solution to be bounded and on proceeding as before, formula (2) may be easily transformed to

$$\lambda_1\chi_1(r, \theta) = \frac{\operatorname{cosec}^2 \frac{\phi}{2}}{Mv^2} \cdot \frac{e^{ikr}}{\sqrt{v}\, r} F(r_0) \cdot \quad \dots \quad (3)$$

where

$$F(r_0) = -k' \int_{r_0}^{\infty} \sin k'r\, V(r) r\, dr \quad \dots \quad (3.1)$$

and

$$k' = 2k \sin \frac{\phi}{2}\, ;\; k = \frac{\pi M v}{h} \quad \dots \quad (3.2)$$

and the critical approach (r_0), which must be always positive, is given by

$$r_0 = \frac{\operatorname{cosec}^2 \frac{\phi}{2}}{Mv^2} \cdot 2\rho \left(\sin \frac{\phi}{2} \pm \sin^2 \frac{\phi}{2} \right) G(r_0) \quad \dots \quad (3.3)$$

where

$$G(r_0) = \pm k' \int_{r_0}^{\infty} \sin k'(r - r_0)\, V(r) r\, dr \quad \dots \quad (3.4)$$

in which the + sign stands for repulsive force and − for attraction. Formulæ (3)⟶(3.4) in the C-system are perfectly general, holding for any interaction potential $V(r)$ between two protons.

If the potential is Coulombian and repulsive $V = +\frac{e^2}{r}$ and so we have from (3)⟶(3.4)

$$\lambda_1\chi_1(r, \theta) = -\frac{e^2}{Mv^2} \operatorname{cosec}^2 \frac{\phi}{2} \cos k'r_0 \cdot \frac{e^{ikr}}{\sqrt{v}\, r} \quad \dots \quad (4)$$

and

$$r_0 = 2\rho \frac{e^2}{Mv^2} \left(\operatorname{cosec} \frac{\phi}{2} + 1 \right);\; \rho = 1.35 \,. \quad \dots \quad (4.1)$$

If, again, the short range interaction is attractive and of Yukawa type, we have for the scattering function and the critical approach

$$\lambda_1\chi_1(r, \theta) = \frac{4\pi^2 MA}{h^2} \cdot \frac{e^{-ar_0}\left\{ \cos k'r_0 + \frac{a}{k'} \sin k'r_0 \right\}}{a^2 + k'^2} \cdot \frac{e^{ikr}}{\sqrt{v}\, r} \quad \dots \quad (5)$$

and
$$r_0 = 2.7 \times \frac{\frac{4\pi^2 MA}{h^2} e^{-\alpha r_0}}{\alpha^2 + k'^2} \left\{ \sin\frac{\phi}{2} - \sin^2\frac{\phi}{2} \right\}. \qquad \ldots \quad (5.1)$$

As has been already noted, eqs. (4)⟶(5.1) are in C-system. For experimental verification we obviously require them to be referred to the laboratory coordinates. In a previous paper of mine [7] I have elaborately discussed this question and have shown that the above formulæ can easily be transformed into the laboratory co-ordinates by simply putting 2θ for ϕ, θ being the laboratory angle co-ordinate.

Now, in case of proton-proton interaction the potential is partly Coulombian repulsive and partly attractive of Yukawa type. Thus the total scattering function is obtained by adding (4) and (5) and we have in L-co-ordinates

$$\lambda_1 \chi_1 (r, \theta) = -\frac{e^2}{Mv^2} \cdot \frac{e^{ikr}}{\sqrt{v}\, r} \left\{ \operatorname{cosec}^2\theta \cos k' r_0 - \frac{Mv^2}{e^2} g_1 f_1 \right\} \qquad \ldots \quad (6)$$

where
$$g_1 = \frac{4\pi^2 MA}{h^2} \cdot \frac{e^{-\alpha r_0'}}{\alpha^2 + k'^2}, \qquad \ldots \quad (6.1)$$

$$f_1 = \cos k' r_0' + \frac{\alpha}{k'} \sin k' r_0', \qquad \ldots \quad (6.2)$$

$$k' = 2k \sin\theta \ ; \ k = \frac{\pi M v}{h}, \qquad \ldots \quad (6.3)$$

and the total value of the critical approach is

$$r_0' = 2.7 \times \frac{e^2}{Mv^2} (\operatorname{cosec}\theta + 1) + 2.7 \times (\sin\theta - \sin^2\theta) g_1. \qquad \ldots \quad (6.4)$$

In the case of proton-proton interaction it is not possible to distinguish between the scattering and the scattered proton ; so it is essential that the effect of exchange should be considered in calculating the intensity of scattering. The scattering function after exchange is obtained by simply putting $\left(\frac{\pi}{2} - \theta\right)$ for θ in eqs. (6)→(6.4). Thus we have after exchange

$$\lambda_1 \chi_1 (r, \theta) = -\frac{e^2}{Mv^2} \cdot \frac{e^{ikr}}{\sqrt{v}\, r} (\sec^2\theta \cos k'' r_0'' - \frac{Mv^2}{e^2} g_2 f_2) \qquad \ldots \quad (7)$$

where
$$g_2 = \frac{4\pi^2 MA}{h^2} \cdot \frac{e^{-\alpha r_0''}}{\alpha^2 + k''^2}, \qquad \ldots \quad (7.1)$$

$$f_2 = \cos k'' r_0'' + \frac{\alpha}{k''} \sin k'' r_0'', \qquad \ldots \quad (7.2)$$

$$k'' = 2k \cos\theta, \qquad \ldots \quad (7.3)$$

and
$$r_0''=2.7\times\frac{e^2}{Mv^2}(\sec\theta+1)+2.7\times(\cos\theta-\cos^2\theta)g_2\,. \quad \ldots \quad (7.4)$$

When the interacting protons have anti-parallel spin, an exchange of their positions would give a new configuration and consequently the total probability is obtained by adding the components. Or, in other words, the total space function is symmetrical. Hence, in this case, we have from (6) and (7) for the total scattering function

$$\lambda_1\chi_1(r,\theta)=-\frac{e^2}{Mv^2}\cdot\frac{e^{ikr}}{\sqrt{v}\,r}\left\{\operatorname{cosec}^2\theta\cos k'r_0'+\sec^2\theta\cos k''r_0''-\frac{Mv^2}{e^2}(g_1f_1+g_2f_2)\right\} \quad \ldots \quad (8)$$

In the case of parallel spin, no new configuration is obtained by exchange. So we have to take the difference of the components in getting the total probability. Or, in other words, the total space function should be taken anti-symmetrical. Hence we have from (6) and (7)

$$\lambda_1\chi_1(r,\theta)=-\frac{e^2}{Mv^2}\cdot\frac{e^{ikr}}{\sqrt{v}\,r}\left\{\operatorname{cosec}^2\theta\cos k'r_0'-\sec^2\theta\cos k''r_0''-\frac{Mv^2}{e^2}(g_1f_1-g_2f_2)\right\}. \quad \ldots \quad (9)$$

The relative intensities in the two cases are respectively

$$I_1=\left(\frac{e^2}{Mv^2}\right)^2\Bigg[\operatorname{cosec}^4\theta\cos^2k'r_0'+\sec^4\theta\cos^2k''r_0''+2\operatorname{cosec}^2\theta\sec^2\theta\cos k'r_0'\cos k''r_0''$$
$$-2(g_1f_1+g_2f_2)\frac{Mv^2}{e^2}(\operatorname{cosec}^2\theta\cos k'r_0'+\sec^2\theta\cos k''r_0'')+\left\{\frac{Mv^2}{e^2}(g_1f_1+g_2f_2)\right\}^2\Bigg]4\cos\theta \quad \ldots \quad (10)$$

and

$$I_2=\left(\frac{e^2}{Mv^2}\right)^2\Bigg[\operatorname{cosec}^4\theta\cos^2k'r_0'+\sec^4\theta\cos^2k''r_0''-2\operatorname{cosec}^2\theta\sec^2\theta\cos k'r_0'\cos k''r_0''$$
$$-2(g_1f_1-g_2f_2)\frac{Mv^2}{e^2}(\operatorname{cosec}^2\theta\cos k'r_0'-\sec^2\theta\cos k''r_0'')+\left\{\frac{Mv^2}{e^2}(g_1f_1-g_2f_2)\right\}^2\Bigg]4\cos\theta. \quad \ldots \quad (11)$$

Now, as the weights of the two events are as 1 : 3, we have for the total intensity $I=\frac{1}{4}I_1+\frac{3}{4}I_2$. Or, we have

$$I=\left(\frac{e^2}{Mv^2}\right)^2\Bigg[\operatorname{cosec}^4\theta\cos^2k'r_0'+\sec^4\theta\cos^2k''r_0''-\operatorname{cosec}^2\theta\sec^2\theta\cos k'r_0'\cos k''r_0''$$
$$-2\frac{Mv^2}{e^2}\left\{(g_1f_1-\tfrac{1}{2}g_2f_2)\operatorname{cosec}^2\theta\cos k'r_0'+(g_2f_2-\tfrac{1}{2}g_1f_1)\sec^2\theta\cos k''r_0''\right\}$$
$$+(g_1^2f_1^2+g_2^2f_2^2-g_1g_2f_1f_2)\left(\frac{Mv^2}{e^2}\right)^2\Bigg]4\cos\theta. \quad \ldots \quad (12)$$

It is a perfectly general formula giving the relative intensity of proton scattering in the Laboratory System at any angle. It should be noted that for $\theta=45°$ the formula is considerably simplified, since at this angle $k'=k''$, $r_0'=r_0''$, $g_1=g_2$ and $f_1=f_2$.

It is evident that the corresponding formula, if the force is only Coulombian, is obtained from (12) by simply putting $g_1=g_2=0$. We have then

$$I_0=\left(\frac{e^2}{Mv^2}\right)^2\left[\operatorname{cosec}^4\theta\cos^2 k'r_0'+\sec^4\theta\cos^2 k''r_0''-\operatorname{cosec}^2\theta\sec^2\theta\cos k'r_0'\cos k''r_0''\right]\times 4\cos\theta. \quad \dots \quad (12.1)$$

Thus the departure from Coulomb scattering is obtained by taking the ratio, *viz.*, $D=I/I_0$. This should give the correct value of the departure. But, unfortunately, the experimenters[8] often measure the departure from Mott's extension of Born-Rutherford formula which has been already shown to be wrong[9] in so far as it has neglected the effect of the critical approach. This extended Born-Rutherford formula for exchange derived by Mott[10] and also in a different way by Kar and Basu[11] is known as the MKB-formula. It may also be deduced from (12.1) by neglecting the critical approach, *i.e.*, by putting $r_0'=r_0''=0$. Thus we have

$$I_{MKB}=\left(\frac{e^2}{Mv^2}\right)^2\left[\operatorname{cosec}^4\theta+\sec^4\theta-\operatorname{cosec}^2\theta\sec^2\theta\right]4\cos\theta. \quad \dots \quad (12.2)$$

Hence the experimentally measured departure becomes

$$D=\frac{I}{I_{MKB}}=\frac{\begin{array}{l}\operatorname{cosec}^4\theta\cos^2 k'r_0'+\sec^4\theta\cos^2 k''r_0''-\operatorname{cosec}^2\theta\sec^2\theta\cos k'r_0'\cos k''r_0''\\ -2\frac{Mv^2}{e^2}\{(g_1f_1-\frac{1}{2}g_2f_2)\operatorname{cosec}^2\theta\cos k'r_0'+(g_2f_2-\frac{1}{2}g_1f_1)\sec^2\theta\cos k''r_0''\}\\ \qquad +(g_1^2f_1^2+g_2^2f_2^2-g_1g_2f_1f_2)\left(\frac{Mv^2}{e^2}\right)^2\end{array}}{\operatorname{cosec}^4\theta+\sec^4\theta-\operatorname{cosec}^2\theta\sec^2\theta}.$$

In order to calculate the value of D for any angle one must know the values of g_1, g_2, f_1, f_2, r_0' and r_0'' for that angle. These may be evaluated easily from $(6.1)\longrightarrow(6.4)$ and $(7.1)\longrightarrow(7.4)$, if the two unknown parameters Λ and a of Yukawa potential are known beforehand. The two unknowns may, however, be evaluated by solving the two equations obtained from (13) by substituting the experimental values of D for any two angles of scattering. But the method is very tedious. A much simpler method would be as follows :—

As already pointed out, formula (13) reduces to a very simple form at $\theta=45°$ and we have

$$D_{45°}=\frac{4\cos^2 k'r_0'-4\cos k'r_0'\cdot\frac{Mv^2}{e^2}g_1f_1+\left(\frac{Mv^2}{e^2}\right)^2g_1^2f_1^2}{4} \quad \dots \quad (13.1)$$

$$=\frac{4\cos^2 k'r_0'-4\cos k'r_0'\,x+x^2}{4}, \quad \dots \quad (13.2)$$

where $x = \frac{Mv^2}{e^2} g_1 / f_1$. By judicious selection we take a trial value of g_1. On substituting this value in (6.4) we get the value of r_0' for a given incident velocity (v). Again from (6.3) k' is known Thus cos $k'r_0'$ and $\cos^2 k'r_0'$ are calculated. Also D_{45} is known from the experiment of Heydenburg and others. Hence on solving (13.2) we get the value of x Thus we calculate the corresponding value of f_1 and so from (6.2) the value of a the unknown parameter in the exponential. As a and g_1 are known, we are now in a position to calculate A from (6.1). Thus from one trial value of g_1 we are able to calculate a set of values of a and A. We next use the above known values of a and A to calculate D_{20} at $\theta = 20°$. If the value of D_{20} so obtained does not agree with the experimental value, the trial value of g_1 is slightly changed. In this way two or three trials help us to find the exact values of g_1, a and A. Graphical method is extremely useful in getting at the above exact values. Now, having got the correct values of a and A we calculate the departure (D) for any angle.

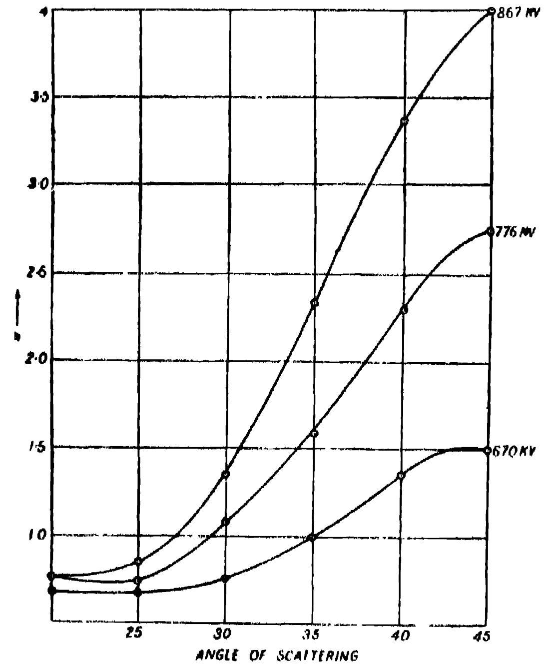

FIG. 1
Variation of D with the Angle Scattering

In Fig. 1 the theoretical curves are drawn giving the variation of D with the angle of scattering, for incident velocities 867 K.V., 776 K.V. and 670 K.V. The experimental points are indicated by solid circles. It is evident that the agreement is so close that no separate experimental curve need be drawn.

It has already been pointed out that $\sqrt{A}$ has the dimension of electrical charge. Therefore $\sqrt{A}/e$ is the charge in electronic unit which gives rise to the short range force. In the following table are given the values of a and $\sqrt{A}/e$ obtained from the curves in Fig. 1.

TABLE I

Incident Velocity	$a \times 10^{-13}$	$A^{\frac{1}{2}}/e$
867 K. V.	0.2839	6.013
776 K. V.	0.2966	6.542
670 K. V.	0.2794	6.190

Thus the charge causing the short range force is found to be $\pm$ $6e$. This may be taken as the charge of the Yukawa particles inside the proton. It is interesting to note that the charge of the Yukawa particles as calculated by Heitler[12] from the mass defect of deuteron lies between $5e$ and $7e$. Thus the present method of calculating the Yukawa charge is more reliable and accurate. Furthermore, according to Heitler, ^{1} $\sim 3 \times 10^{-13}$ whereas from the above table the mean value of ^{1} is $3.5 \times 10^{-}$ The above striking agreement suggests that the present method when extended to other interactions may lead to a general method of determining the short range charge.

As, in the present wave-statistical method, the determination of the critical approach is very important, it is advisable to discuss the variation of the critical approach with angle and how it is affected by the presence of the short range attractive force. This variation, as calculated from formula (6.4), is shown in Fig. 2. The curves A, B and C indicate the variation of the total critical

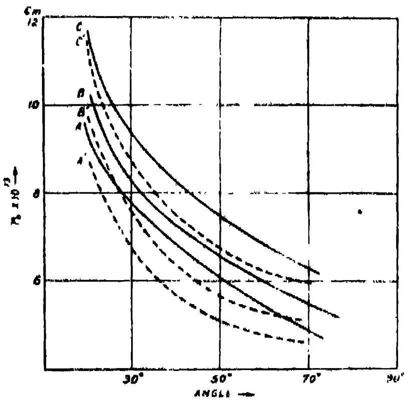

FIG. 2

Variation of Critical Approach with Angle approach for incident velocities 867 K V., 776 K.V. and 670 K.V. whereas the curves A′, B′ and C′ give the corresponding variation of the critical approach due to the repulsive force only. On comparing the corresponding curves A, A′, etc., it is evident that the effect of the short range attractive force is to slightly increase the value of the critical approach, which is, however, negligible at small angles.

Before concluding I would like to make a few remarks on the charge and mass of the Yukawa particles giving rise to the short range force between the protons. This force between neutron and proton or between two protons is found to be always attractive. It strongly suggests that the charges causing this short range force are opposite in nature and are developed temporarily through polarisation during the process of scattering, specially when the particles are at small distances apart. For, if the charges $\pm 6e$ permanently existed within the protons and neutrons, it would have been certainly possible to observe cases of short range repulsion between neutron-proton and proton-proton. Now, the reason why the law of short range potential is different from Coulombian appears to me to be only due to the difference in the distribution of charge It is well known that for uniform distribution of charge on a sphere the potential is Coulombian. So it is quite likely that for some special distribution the potential may be of Yukawa type This is all the more plausible in view of the fact that the dimensions of the two charges are the same.

Now, as the short range charges associated with Yukawa particles in case of neutron-proton and proton-proton interactions are the same ($\pm 6e$), the masses of these particles must also be identical. It may be calculated in a very simple way. As the light quanta arising from the electromagnetic field satisfy the potential equation of the type

$$\Delta V - \frac{1}{c^2}\frac{d^2V}{dt^2} = 0, \qquad \ldots \quad (14)$$

the corresponding equation for the Yukawa potential will be of the type

$$\Delta V + a^2 V - \frac{1}{c^2}\frac{d^2V}{dt^2} = 0. \qquad \ldots \quad (14.1)$$

Eq.(14.1) may be regarded as a wave equation for a particle with a rest mass

$$m = \frac{ah}{2\pi c}. \qquad \ldots \quad (14.2)$$

Taking the mean value of a to be 0.2866×10^{13} (*vide* Table I), we find from (14.2), for the mass of Yukawa particles,

$$m \sim 110.8 \text{ electron mass.}$$

From the saturation properties of the nuclear forces, we know that the forces between a neutron and a proton are of exchange type. Thus the proton gives its long range positive charge to the neutron, thereby converting it into a proton, while becoming itself a neutron. A similar type of exchange cannot obviously take place in proton-proton interaction, since both the particles have the same long range charge. Therefore, the Yukawa particles giving rise to the short range force in proton-proton interaction should possess no long range electrical charge. These neutral particles are called 'neutrettos.' Thus there are two classes of Yukawa particles having the same mass (110.8 e.m.) and same short range charge ($\pm 6e$), *viz.*, (*i*) mesotrons, having long range charge e and taking part in neutron-proton interaction, and (*ii*) neutrettos, having no long range charge and taking part in proton-proton interaction.

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